Simple Correlation for the Rise in Free Surface Caused by Submerged Jets Directed Upward

M. A. Delichatsios
Factory Mutual Research Corporation
Norwood, MA 02062

A simple correlation is presented for the free surface rise caused by submerged jets; the correlation was verified by using data from Narusawa et al. (1983). The proposed correlation is extended to buoyant flows intercepted by a density interface.

In a recent paper, Narusawa et al. (1983) presented careful measurements of the free surface rise caused by submerged jets. Figure 1 is a sketch of the flow configuration, which includes a nozzle submerged in water that discharges water vertically toward the free surface. The distance of the nozzle from the free surface is denoted by H and the free surface rise by h. The discharged fluid has the same density as the ambient fluid and a total momentum flow rate, $\dot{M}_o$. The figure also shows the nozzle diameter, d_o, the lateral extent of the jet, b, and the lateral extent of the surface rise, r_s, at the level of free surface. As soon as the jet reaches the free surface, the free surface rises and then folds back because gravity decelerates the flow emerging into the air. One observes a similar flow in a buoyant jet intercepted by an interface that separates fluids having different temperatures. Such flows may develop when thermal plumes rise in the atmosphere, or when fluids or different temperatures interact (as in solar energy storage tanks or cooling water discharge from power plants).

Narusawa et al. correlated their results in a rather complex way by using classical integral models for forced turbulent jets (Turner, 1973). Starting from a global dimensional analysis of the flow, we propose here a simpler correlation of those experimental results. This new correlation can be applied also to similar flow situations such as those cited previously.

The basic assumptions of our hypothesis are:

1. The flow is a fully developed turbulent flow.
2. The nozzle diameter, d_o, and hence the initial flow rate, do not significantly affect the free surface rise.

Starting from these assumptions, one concludes that the controlling independent parameters are the fluid density, ρ_f, the jet momentum flow rate, $\dot{M}_o$, the fluid depth, H, from the nozzle and the effective gravitational acceleration, $[(\rho_f - \rho_a)/\rho_f]\, g \equiv g'$, wherein ρ_a is the air density. The dependent variable is the free surface rise, h. Dimensional analysis allows the following functional relation between the flow parameters:

$$\frac{h}{H} = \text{fcn}\left[\left(\frac{\dot{M}_o/\rho_f}{g'}\right)^{1/3}/H\right] \tag{1}$$

A test of this relation is shown in Figure 1, which includes data taken from Figure 2 of the Narasawa et al. paper. The ordinate is the ratio of surface rise to the depth of the fluid, h/H, and the abscissa is the ratio of a length characteristic of the flow, $(\dot{M}_o/\rho_f)^{1/3}/g'$, to the depth of the fluid, H. The abscissa is related to the test parameters used by Narusawa et al. by the following equation:

$$\frac{1}{H}\left(\frac{4\,\dot{M}_o/\rho_f}{\pi g'}\right)^{1/3} = \frac{d_o}{H}\left(\frac{U_o^2}{d_o g'}\right)^{1/3} = \frac{d_o}{H}\,\text{Fr}_N^{2/3}. \tag{2}$$

since the momentum flow rate, $\dot{M}_o = \pi\rho_f(d_o^2/4)\,U_o^2$ and $g' \simeq g$ for a liquid-air interface where $\rho_f \gg \rho_a$; $Fr_N^2 = U_o^2/d_o g'$ is a Froude number based on the nozzle diameter (d_o is the nozzle diameter and U_o the exit velocity at the nozzle).

It is remarkable that all the data are correlated well, Figure 2, except for the data corresponding to Froude number $Fr_N = 2.89$ and nozzle diameter $d_o = 7.8$ mm (solid triangle symbols). We ascribe this discrepancy to the relatively low Reynolds number of $U_o d_o/\nu_f = 6{,}250$ for this flow and to the small ratio of the fluid depth over the nozzle diameter $H/d_o < 15$. Under these conditions, the jet flow does not have time to develop to a fully turbulent flow. A best fit of the data (excluding the results for $d_o = 7.8$ mm, $Fr_N = 2.89$) provides the following relation for the free surface rise:

$$\frac{h}{H} = 3.7\left[\frac{4(\dot{M}_o/\rho_f)}{\pi g H^3}\right]^{2/3}. \tag{3}$$

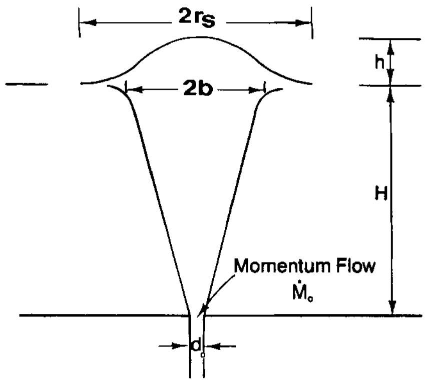

Figure 1. Flow configuration in a water jet submerged in a water pool of height *H*.

It is difficult to specify exactly the flow conditions for which Eq. 3 breaks down, since no sufficient experimental data exist. We expect that Eq. 3 will be applicable if the major part of the submerged jet is fully turbulent, and if the jet does not emerge from the free surface as a water jet through the air. Based on the experimental test conditions (Narusawa et al.), we expect that Eq. 3 will be applicable if: $H/d_o > 10$, $R_e = U_o d_o/\nu_f > 6{,}000$, and $\{[(4/\pi)\,(\dot{M}_o/\rho_f)/g]^{1/3}/H\} < 1.7$.

Although dimensional analysis provides a correlation between the flow parameters, Eq. 3, it does not give insight into a physical understanding of the problem. We have developed a simple physical model for explaining the semiempirical relation for the free surface rise, Eq. 3. The shape of the surface rise is characterized by its maximum height h and the radial distance, r_s, where the free surface becomes horizontal (Figure 1). We first observe that the vertical momentum of the jet must be equal to the weight of the fluid that rises above the horizontal free surface:

$$\dot{M}_o/\rho_f \sim r_s^2\, hg'. \tag{4}$$

Next, we note that the residence time of the fluid in the elevated volume must be equal to a characteristic flow time in the elevated volume:

$$\frac{r_s^2 h}{ub^2} = \frac{h}{u_*} \tag{5}$$

where b is the width of the jet at the free surface elevation and u_* is a characteristic velocity inside the elevated volume.

Finally, we may determine the characteristic velocity, u_*, by observing that the rate of change in the potential energy of the jet volume, as it tends to emerge at the free surface, is equal to the flow of kinetic energy in the turbulent flow inside the elevated volume. The potential energy of the jet volume is:

$$mg'H \sim \rho_f g' H^2 b^2 \tag{6a}$$

The kinetic energy is

$$\tfrac{1}{2}\,\rho_f u^{*2}\, u^* b^2 = \rho_f\, \tfrac{1}{2}\, u^{*3} b^2. \tag{6b}$$

Therefore:

$$\frac{d}{dt}(mg'H) \sim \rho_f u^{*3} b^2 \tag{6c}$$

or

$$g'Hb^2 \frac{dH}{dt} \sim u^{*3} b^2 \tag{6d}$$

or

$$u^{*3} \sim g'Hu \tag{6e}$$

where u is the mean jet velocity at the free surface elevation, which is equal to the virtual rate of change of the highest elevation, H, of the impinging jet ($u = dH/dt$).

By combining Eqs. 4, 5, and 6c, we obtain:

$$\frac{h}{H} \sim \left[\frac{(\dot{M}_o/\rho_f)}{g'}\right]^{2/3} \frac{1}{H^2} \tag{7}$$

which is in agreement with the empirical relation shown by Eq. 3.

We may extend Eq. 3 to other flow situations for a fluid impinging on a density interface. It is convenient first to cast Eq. 3 in terms of properties of the jet flow at the free surface elevation. Since $b = 0.16\,H$ and $M_o = \rho_f \pi\, b^2 u^2$ (top hat profiles, Turner, 1973), Eq. 3 takes the form

$$\frac{h}{b} = 1.5\left[\frac{u^2}{bg'}\right]^{2/3} \tag{8}$$

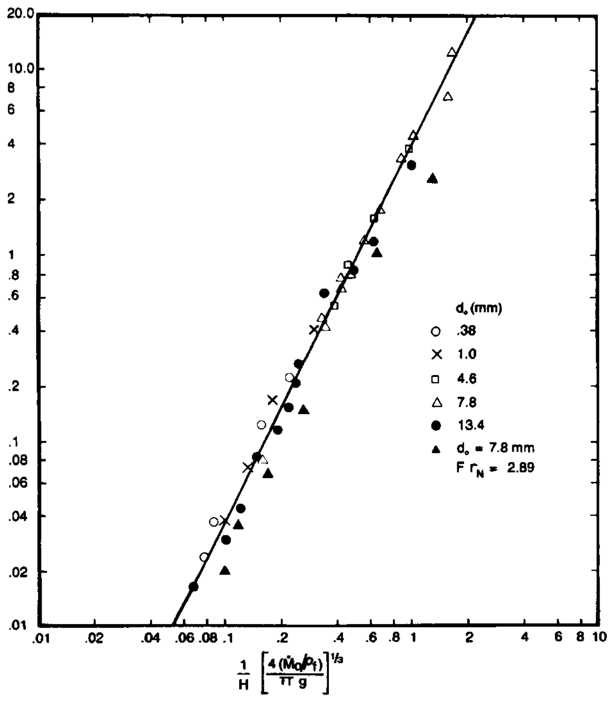

Figure 2. Surface rise *h* produced by a water jet with momentum $\dot{M}_o$, submerged in a water pool of height *H*. d_o, nozzle dia. Fr_N, Froude number at nozzle.

where u is the mean jet velocity at the interface; b is the jet radius at the interface; our $g' = \rho_f - \rho_a/\rho_f$ is the density defect ratio between the jet fluid ρ_f and the fluid beyond the interface. (Top hat profiles are assumed.)

Equation 8 could be used for estimating the penetration depth of a liquid jet directed downward through air into a heavier fluid, as well as for predicting the interface rise of a thermal plume that encounters a dense environment at a given elevation from its inception. Mixing and entrainment will certainly modify the coefficient in Eq. 8; however, we emphasize that Eq. 8 is applicable only (Figure 1) if the impinging flow does not penetrate through the interface to form a distinct jet (fountain flow).

In the case of fountain flow, the classical result (Turner, 1973) for turbulent negatively buoyant plumes is applicable for the penetration height before the upward flow stops, Figure 3:

$$h \sim \frac{\dot{M}_o^{3/4}}{F_o^{1/2}} \tag{9}$$

where $\dot{M}_o$ is the momentum flow rate ($\sim U_o^2 d_o^2$) and $-F_o$ is the negative buoyancy flow at the source ($\sim U_o d_o^2 g_o'$). Equation 9 may be derived by dimensional analysis or by using the physical model that we have presented in this work. Specifically, in these circumstances (Figure 3) the lateral flow extent is proportional to the height h, while the characteristic velocity is the centerline plume velocity. By momentum balancing we obtain

$$h^3 g' \sim \dot{M}_o \tag{10a}$$

and by energy balancing we obtain

$$u^3 \sim u^{*3} \sim g' h u \tag{10b}$$

Finally, conservation of buoyancy gives

$$h^2 u g' \sim F_o \tag{10c}$$

The penetration height, Eq. 9 can readily be obtained by using Eq. 10a, b, c. For comparison with Eq. 8, the penetration height for negatively buoyant plumes is written in terms of the source radius b_o, the initial density defect g_o', and the initial velocity u_o

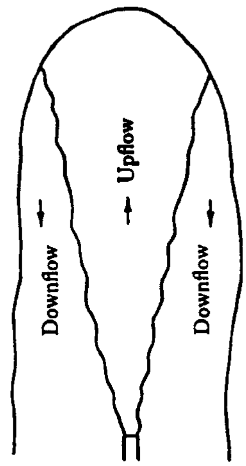

Figure 3. Flow regions in a heavy salt jet projected upward in water (Turner, 1973, p. 175).

$$\frac{h}{b_o} = 2.46 \left(\frac{u_o^2}{b_o g_o'}\right)^{1/2} \tag{11}$$

where the coefficient has been obtained from experimental data (Turner, 1973).

We have demonstrated in this work that the free surface rise of a turbulent jet impinging upward at a density interface can be predicted by a simple relation suggested by dimensional analysis and explained by a simple physical model. The proposed expression is applicable when no fountain flow (no distinct jet) develops inside the host fluid; in the case of fountain flow, Eq. 11 is applicable.

Literature Cited

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